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NCF-003-1162004

Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

April / May – 2017

Mathematics : 2004

(Methods in Partial Differential Equation)

[New Course]

Faculty Code : 003
Subject Code : 1162004

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (i) There are **five** questions.
(ii) **All** questions are **compulsory**.
(iii) **Each** question carries **14** marks.

1 Do as directed : (Each question carries two marks) **14**

(a) Solve the equation $\left(2D'+D^2\right)z=0$.

(b) State the general form for the P.D.E. when reduces to the canonical transformation for

(i) Hyperbolic and

(ii) Elliptic.

(c) Find the direction cosines of the normal to the surface $2x+3y+5z=7$ at the point (1,0,1).

(d) Find the complete integral of $p+q=pq$.

(e) State Lipchitz condition for the functions of three variables (x,y,z) from the point (a,b,c).

(f) Verify the equation is exact or not $(6x+yz)dx+(xz-2y)dy+(2z+xy)dz=0$.

(g) Find the partial differential equation for $z=f\left(x^2-y\right)+g\left(x^2+y\right)$ where f and g are arbitrary functions.

2 Answer any two of the following : **2×7=14**

(a) Prove that for any non zero functions $X = (P, Q, R)$ where P, Q, R are the functions of x, y, z then $X \cdot Curl\ X = 0$ if $Pdx + Qdy + Rdz = 0$ is integrable.

(b) Find the surface which is orthogonal to the one parameter system, $z = cxy\left(x^2 + y^2\right)$ which passes through the circle $x^2 - y^2 = A^2,\ z = 0$.

(c) Prove that a pfaffian differential equation

$$\left(y^2 + yz + z^2\right)dx + \left(x^2 + xz + z^2\right)dy = -\left(x^2 + yx + y^2\right)dz$$

is integrable. Also find the complete primitive.

3 All are compulsory : **14**

(a) Verify that $F(D, D')\left[e^{ax+by}\right] = F(a,b)e^{ax+by}$. **4**

(b) (i) Find the integral curves of the equation **5**

$$\frac{dx}{(xz - y)} = \frac{dy}{(yz - x)} = \frac{dz}{\left(1 - z^2\right)}.$$

(ii) Find the integral curves of the equation

$$\frac{dx}{x^2\left(y^3 - z^3\right)} = \frac{dy}{y^2\left(z^3 - x^3\right)} = \frac{dz}{z^2\left(x^3 - y^3\right)}.$$

(c) Find the complete integral of $\left(p^2 + q^2\right)x = pz$. **5**

OR

3 All are compulsory : **14**

(a) Solve the equation : **4**

(i) $(y-z)dx+dy-dz=0$.

(ii) $\left(y^2+z^2\right)dx+xydy+xzdz=0$.

(b) Using Natani's method **5**

$2y(a-x)dx+\left(z-y^2+(a-x)^2\right)dy-ydz=0$.

(c) Prove that **5**

$F(D,D')\left[e^{ax+by}\cdot h(x,y)\right]=e^{ax+by}F(D+a,D'+b)\left[h(x,y)\right]$.

4 Answer any two of the following : **2×7=14**

(a) Find the equation of integral surface of the differential equation $(x-y)y^2p+(y-x)x^2q=\left(x^2+y^2\right)z$ passses through $y=0$ and $xz=a^3$.

(b) Find the G.S. of

$\left(D^3-4D^2D'+4DD'^2\right)z=\cos(2x-y)e^{2x-y}+e^{x+y}\left(x-y^2\right)$.

(c) (i) Find the solution of $\left(x^2D^2+2xyDD'-xD\right)z=\dfrac{x^3+y^3}{xy}$.

(ii) Using Jacobi's method solve $xyp=q$.

5 Answer any two of the following : **2×7=14**

(a) Solve $2z + p^2 + qy + 2y^2 = 0$.

(b) If $(\alpha + \beta D' + \gamma)^n$ with $\alpha \neq 0$ is a factor of $F(D, D')$, then a solution of the equation $F(D, D')$ is,

$$z = e^{-\frac{\gamma}{\alpha}x}(\varnothing_1(\beta x - \alpha y) + x\varnothing_2(\beta x - \alpha y) + x^2\varnothing_3(\beta x - \alpha y) + \ldots\ldots\ldots + x^{n-1}\varnothing_n(\beta x - \alpha y))$$

where $\varnothing_i = \varnothing_i(\varepsilon)$ is an arbitrary function of a single variable $(i = 1, 2, \ldots.., n)$.

(c) Classify the equation and convert it into canonical form $y^2 r + 4x^2 t = xy. (x \neq y \neq 0)$.
